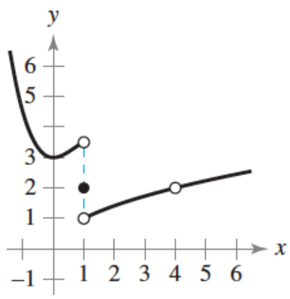
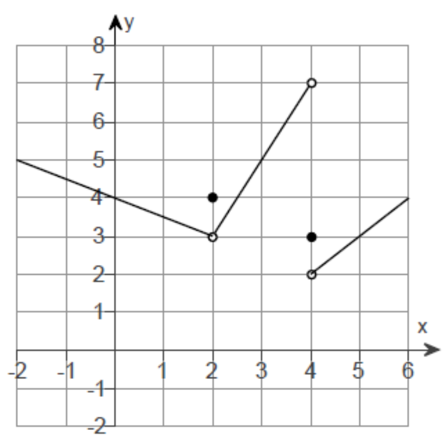
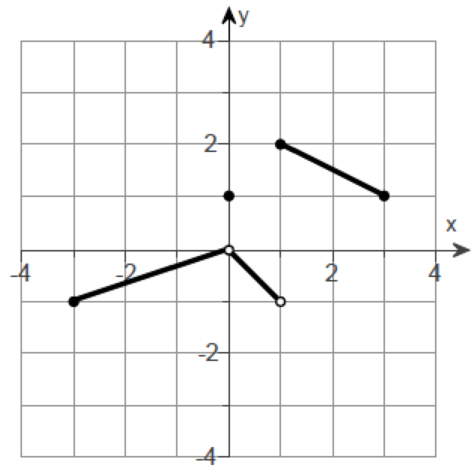
AP Calculus AB

Unit 2 – Limits and Continuity

There are no great limits to growth because there are no limits of human intelligence, imagination, and wonder.
– Ronald Reagan

Answer the following questions.

1	For the function $f(x)=5x^2$, as the x-value gets closer and closer to 3, $f(x)$ gets closer and closer to what value?
2	For the function $f(x)=\dfrac{x^2-4}{x-2}$, as the x-value gets closer and closer to 2, $f(x)$ gets closer and closer to what value?
3	For the function $f(x)=e^x+1$, as the x-value gets closer and closer to 0, $f(x)$ gets closer and closer to what value?
4	The graph of $f(x)$ is given below, use the graph to answer the following questions.  a) $\lim\limits_{x\to 4^-} f(x)$ b) $\lim\limits_{x\to 4^+} f(x)$ c) $\lim\limits_{x\to 4} f(x)$ d) $f(4)$ e) $\lim\limits_{x\to 1^-} f(x)$ f) $\lim\limits_{x\to 1^+} f(x)$ g) $\lim\limits_{x\to 1} f(x)$ h) $f(1)$
5	Use the graph of $f(x)$ to estimate the limits and value of the function, or explain why the limit does not exist.  a) $\lim\limits_{x\to 4^-} f(x)$ b) $\lim\limits_{x\to 4^+} f(x)$ c) $\lim\limits_{x\to 4} f(x)$ d) $f(4)$ e) $\lim\limits_{x\to 2^-} f(x)$ f) $\lim\limits_{x\to 2^+} f(x)$ g) $\lim\limits_{x\to 2} f(x)$ e) $f(2)$
6	Answer each statement as either True or False. a) If $f(1)=5$, then $\lim\limits_{x\to 1} f(x)$ exists. b) If $f(1)=5$ and $\lim\limits_{x\to 1} f(x)$ exists, then $\lim\limits_{x\to 1} f(x)=5$. c) If $\lim\limits_{x\to 1} f(x)=5$, then $f(1)=5$.

7	 Use the graph of $f(x)$ above to answer each statement as either True or False.

a) $\lim_{x\to 0} f(x)$ exists	b) $\lim_{x\to 0} f(x)=1$	c) $\lim_{x\to 0} f(x)=0$
d) $\lim_{x\to 1} f(x)=2$	e) $\lim_{x\to 1} f(x)=-1$	f) $\lim_{x\to c} f(x)$ exists for every point c in $(-3,1)$.
g) $\lim_{x\to 1} f(x)$ does not exist	h) $f(0)=0$	i) $f(0)=1$
j) $f(1)=-1$	k) $f(1)=2$	l) $\lim_{x\to 2} f(x)$ exists

8	Simplify $\dfrac{x^2+7x+12}{x^2-16}$

Let b and c be real numbers, let n be a positive integer, and let f and g be functions with the following limits:

$$\lim_{x\to c} f(x) = L \quad \text{and} \quad \lim_{x\to c} g(x) = M.$$

- $\lim\limits_{x\to c} k = k$
- $\lim\limits_{x\to c} x = c$
- $\lim\limits_{x\to c}\left[f(x) \pm g(x)\right] = \lim\limits_{x\to c} f(x) \pm \lim\limits_{x\to c} g(x) = L \pm M$
- $\lim\limits_{x\to c}\left[f(x)\cdot g(x)\right] = \lim\limits_{x\to c} f(x)\cdot \lim\limits_{x\to c} g(x) = L\cdot M$
- $\lim\limits_{x\to c}\dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x\to c} f(x)}{\lim\limits_{x\to c} g(x)} = \dfrac{L}{M};\quad M \neq 0$
- $\lim\limits_{x\to c}\left[bf(x)\right] = bL$
- $\lim\limits_{x\to 0}\sqrt[n]{f(x)} = \sqrt[n]{L}$
- $\lim\limits_{x\to 0}\left[f(x)\right]^n = L^n$

1) Suppose $\lim\limits_{x\to c} f(x) = 5$, $\lim\limits_{x\to c} g(x) = -2$, and $\lim\limits_{x\to c} h(x) = 9$, find

a) $\lim\limits_{x\to c}\left[f(x)g(x)\right]$

b) $\lim\limits_{x\to c}\left[f(x) - g(x)\right]$

c) $\lim\limits_{x\to c}\sqrt{h(x)}$

d) $\lim\limits_{x\to c}\left[\dfrac{g(x)+1}{x}\right]$

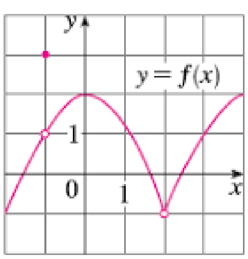
e) $\lim\limits_{x\to c}\left[2h(x) - 3g(x)\right]$

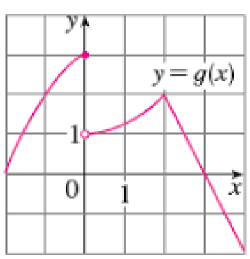
f) $\lim\limits_{x\to c}\left[\dfrac{f(x)}{h(x)}\right]$

g) $\lim\limits_{x\to c}\left[\dfrac{g(x)}{f(x)}\right]$

h) $\lim\limits_{x\to c}\left[g(x)\right]^2$

2) The graphs of f and g are given below. Use the graphs to evaluate each limit.





a) $\lim\limits_{x\to -1}\left[f(x) + g(x)\right]$

b) $\lim\limits_{x\to 3} f\left(g(x)\right)$

c) $\lim\limits_{x\to 3}\left[f(x)g(x)\right]$

d) $\lim\limits_{x\to 2}\left[2f(x) + 5g(x)\right]$

e) $\lim\limits_{x\to -1}\left[\dfrac{f(x)}{g(x)}\right]$

f) $\lim\limits_{x\to 2}\left[xf(x)\right]$

3) Suppose that $\lim_{x \to 7} f(x) = 0$ and $\lim_{x \to 7} g(x) = 5$. Determine the following limits.

a) $\lim_{x \to 7} \left(g(x) + 5 \right)$

b) $\lim_{x \to 7} x f(x)$

c) $\lim_{x \to 7} g^2(x)$

d) $\lim_{x \to 7} \left[\dfrac{g(x)}{f(x) - 1} \right]$

AP Calculus AB – Worksheet 9 **Evaluating Limits Algebraically**

Once we accept our limits, we go beyond them. – Albert Einstein

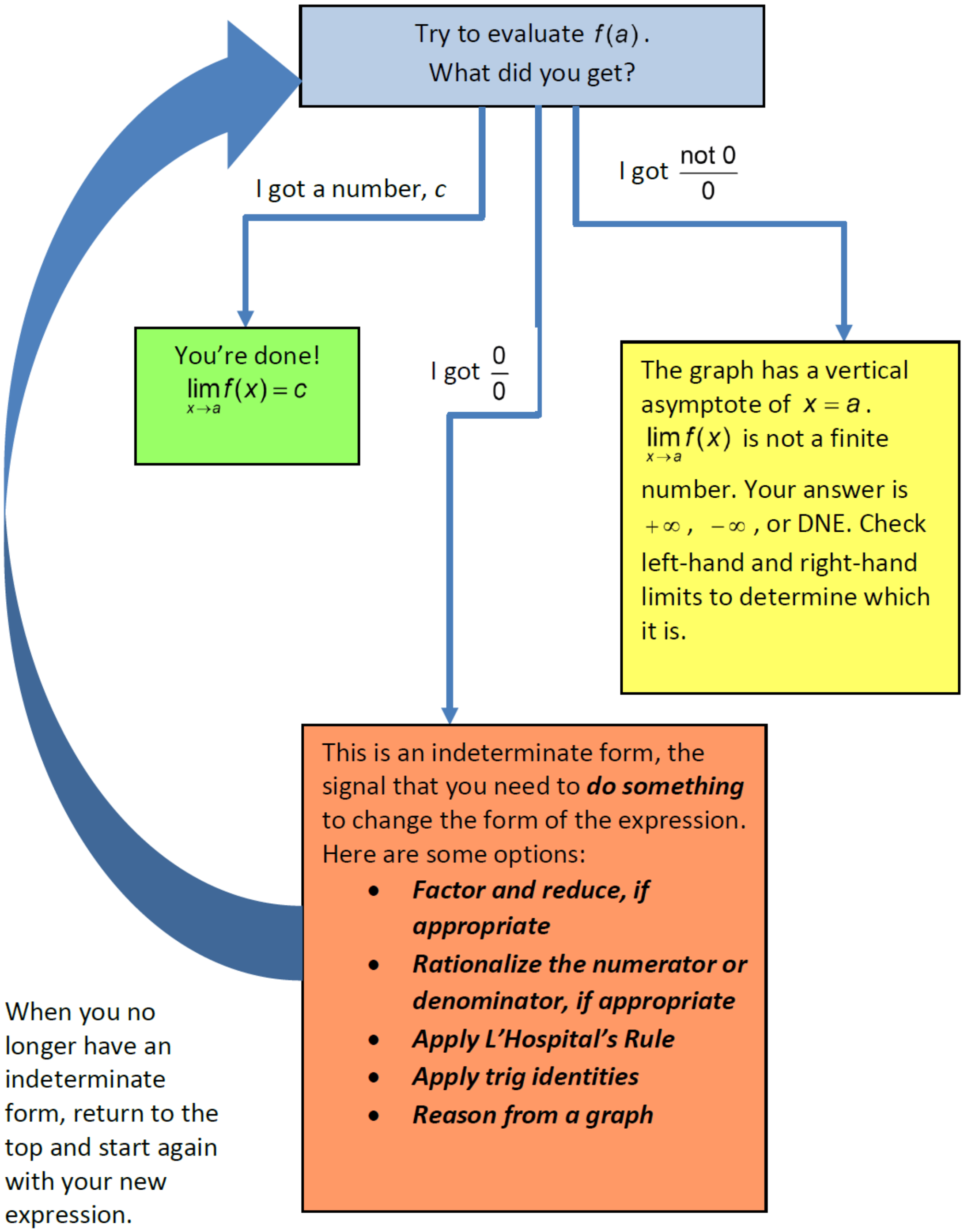
Evaluate the limit.

1	$\lim\limits_{x\to -3}(3x+2)$
2	$\lim\limits_{x\to 1}(3x^3-2x^2+4)$
3	$\lim\limits_{x\to 3}\dfrac{\sqrt{x-1}}{x-4}$
4	$\lim\limits_{x\to 2}\cos\dfrac{\pi x}{3}$
5	$\lim\limits_{x\to 1}\sin\dfrac{\pi x}{2}$
6	$\lim\limits_{x\to 0}\sec 2x$
7	$\lim\limits_{x\to 3}\dfrac{x^2-9}{x+3}$
8	$\lim\limits_{x\to 0}\dfrac{3x^2-2x+1}{x}$
9	$\lim\limits_{x\to 1}\dfrac{5x^2+6x+10}{8x-1}$
10	$\lim\limits_{x\to 0}\dfrac{2+5x+\sin x}{5\cos x}$
11	For the following piecewise function $f(x)=\begin{cases}5-x, & x<3\\ \dfrac{3x}{4}+1, & x>3\end{cases}$, evaluate $\lim\limits_{x\to 3}f(x)$.
	For the following piecewise function: $f(x)=\begin{cases}x+1, & x<2\\ x^2-1, & 2<x<4\\ \sqrt{x+5}, & x\geq 4\end{cases}$, evaluate: 12) a) $\lim\limits_{x\to 2^-}f(x)$ b) $\lim\limits_{x\to 2^+}f(x)$ c) $\lim\limits_{x\to 2}f(x)$ d) $f(2)$ 13) a) $\lim\limits_{x\to 4^-}f(x)$ b) $\lim\limits_{x\to 4^+}f(x)$ c) $\lim\limits_{x\to 4}f(x)$ d) $f(4)$

Limit Strategy Flowchart

The following flowchart can help you pick a strategy for evaluating limits of the form $\lim\limits_{x\to a} f(x)$, where $f(x)$ is a rational expression.

Study the flowchart, making sure you understand it. In your textbook, turn to exercises asking you to evaluate a variety of limit expressions, and practice applying the flowchart.



Evaluate each limit

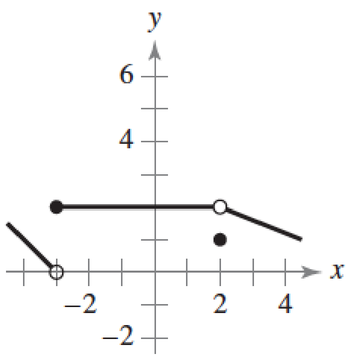
1) $\lim_{x\to 1}\left(12x^3+x^2-1\right)$	2) $\lim_{x\to 5}\frac{x+1}{x+2}$	3) $\lim_{x\to 4}\frac{x^2+5x+4}{x+2}$
4) $\lim_{x\to 0}\frac{x^2-2x}{x}$	5) $\lim_{x\to 4}\frac{4-x}{2-\sqrt{x}}$	6) $\lim_{x\to \frac{\pi}{4}}\left(\sin^2 x+\cos^2 x\right)$
7) $\lim_{x\to -1}\frac{2x^2-x-3}{x+1}$	8) $\lim_{x\to 3}\frac{\sqrt{x+13}-4}{x-3}$	9) $\lim_{x\to -3}\frac{2x+1}{x+3}$
10) $\lim_{x\to -8}\frac{y^2+9y+8}{y^2-64}$	11) $\lim_{x\to 10}\frac{x^2-100}{x-10}$	12) $\lim_{h\to 0}\frac{(9+h)^2-81}{h}$
13) $\lim_{x\to 1} f(x)$ for $f(x)=\begin{cases}2x+1, & x<1\\ x-3, & x\geq 1\end{cases}$	14) Suppose $\lim_{x\to 3} f(x)=-5$ and $\lim_{x\to 3} g(x)=2$, evaluate $\lim_{x\to 3} 4\left[f(x)-2g(x)\right]$	
15) Suppose $f(x)=\begin{cases}x^2-ax, & x\leq 2\\ 3x+6, & x>2\end{cases}$, find the value a that guarantees that $\lim_{x\to 2} f(x)$ exists.		
16) Suppose $f(x)=\begin{cases}x^2-4x, & x<-1\\ ax^3-2, & x>-1\end{cases}$, find the value a that guarantees that $\lim_{x\to -1} f(x)$ exists.		
17) If $\lim_{x\to 7}\frac{f(x)+9}{x-7}=5$, find $\lim_{x\to 7} f(x)$.		
18) If $\lim_{x\to 4}\frac{f(x)-7}{x-4}=6$, find $\lim_{x\to 7} f(x)$.		

AP Calculus AB – Worksheet 11 **Limits – The Difference Quotient/The Squeeze Theorem**

The only limits to the possibilities in your life tomorrow are the "buts" you use today. – Les Brown

For #1-4, find $\lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x)-f(x)}{\Delta x}$.	
1. $f(x)=2x+3$	2. $f(x)=x^2-4x$
3. $f(x)=\dfrac{4}{x}$	4. $f(x)=\sqrt{x}$

Use the graph of $f(x)$ shown below to answer 5-7. The domain of $f(x)$ is $(-\infty,\infty)$.



5. $\lim\limits_{x\to 2} f(x)$

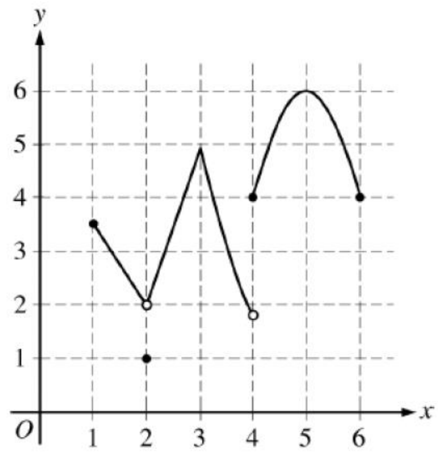
6. $\lim\limits_{x\to -3} f(x)$

7. Identify the values of c for which $\lim\limits_{x\to c} f(x)$ exists. (Hint: think interval)

Find the limit, if it exists.	
8. $\lim\limits_{x\to 1} \dfrac{x^2+3x+2}{x-1}$	9. $\lim\limits_{x\to 5} \dfrac{x-5}{x+1}$
10. $\lim\limits_{x\to 2} \dfrac{x-3}{x^2-25}$	11. $\lim\limits_{x\to \frac{\pi}{6}} \left(\sin^2 x\right)$

12	If $1 \le f(x) \le x^2+2x+2$ for all x, find $\lim\limits_{x\to -1} f(x)$. Justify.
13	If $-3\cos(\pi x) \le f(x) \le x^3+2$, evaluate $\lim\limits_{x\to 1} f(x)$. Justify
14	If $2-x^2 \le g(x) \le 2\cos x$ for all x, find $\lim\limits_{x\to 0} g(x)$.
15	If $\lim\limits_{x\to 4} \dfrac{f(x)-5}{x-2}=1$, find $\lim\limits_{x\to 4} f(x)$.

Review



Graph of f

For questions 1-2, use the graph of the function f shown above.

1	a) $\lim\limits_{x\to 2^-} f(x)$ b) $\lim\limits_{x\to 2^+} f(x)$ c) $\lim\limits_{x\to 2} f(x)$ d) $f(2)$
2	a) $\lim\limits_{x\to 4^-} f(x)$ b) $\lim\limits_{x\to 4^+} f(x)$ c) $\lim\limits_{x\to 4} f(x)$ d) $f(4)$
3	$\lim\limits_{x\to 4}\left(x^2-3x+7\right)=$
4	$\lim\limits_{x\to 1}\sqrt{x^3+7x+1}=$
5	$\lim\limits_{x\to -3}\dfrac{x^2-9}{x^2-2x-15}=$
6	$\lim\limits_{x\to 0}\dfrac{2x^6+6x^3}{4x^5+3x^3}=$
7	$\lim\limits_{x\to 3^-}\dfrac{\lvert x-3\rvert}{x-3}=$
8	Given $f(x)=\begin{cases} x^2\sin(\pi x), & x<2 \\ x^2+cx-18, & x\ge 2 \end{cases}$, find the value of c that $\lim\limits_{x\to 2} f(x)$ exists

Limits at Infinity

Evaluate each limit

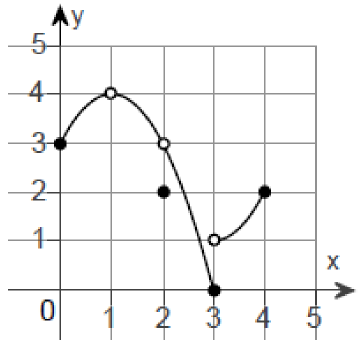
1) $\lim_{x\to\infty}\left(-5x^3-7x^2+1\right)$	2) $\lim_{x\to-\infty}\left(5x^3-7x^2+1\right)$	3) $\lim_{x\to\infty}\left(8+\frac{9}{x^2}\right)$	4) $\lim_{x\to\infty}\frac{2+6x+9x^2}{x^2}$
5) $\lim_{x\to\infty}\frac{9x+8}{9x+7}$	6) $\lim_{x\to-\infty}\frac{9x+8}{9x+7}$	7) $\lim_{x\to\infty}\frac{\sqrt{2x^2-4}}{x+3}$	8) $\lim_{x\to-\infty}\frac{\sqrt{x^2-4}}{x+3}$
9) $\lim_{x\to\infty}\frac{x^3}{e^{3x}}$	10) $\lim_{x\to-\infty}\frac{x^3}{e^{3x}}$	11) $\lim_{x\to\infty}\frac{\sin 3x}{15x}$	12) $\lim_{x\to-\infty}\frac{7-6x+\sin 2x}{6x+\cos 2x}$
13) For which of the following does $\lim_{x\to\infty} f(x)=0$? I. $f(x)=\frac{\ln x}{x^{99}}$ II. $f(x)=\frac{e^x}{\ln x}$ III. $f(x)=\frac{x^{99}}{e^x}$		14) Determine the horizontal asymptote(s) of each of the following. a) $y=\frac{20x^2-x}{1+4x^2}$ b) $f(x)=\frac{(3x+8)(5-4x)}{(2x+1)^2}$ c) $g(x)=\frac{x}{\sqrt{x^2-1}}$	

AP Calculus AB - Worksheet 13 **Continuity**

Strive for continuous improvement, instead of perfection. – Kim Collins

3-Part Definition of Continuity

1. Determine the *x*-values at which the function *f* below has discontinuities. For each value state which condition is violated from the 3-part definition of continuity.



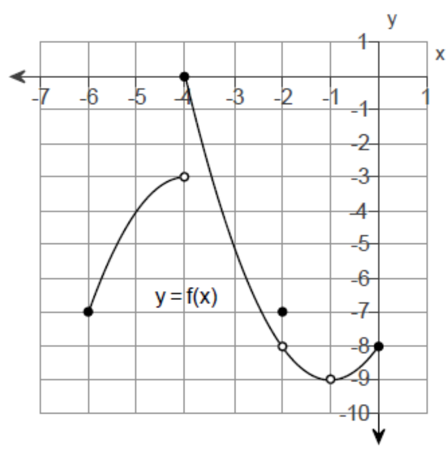
I. $f(c)$ is defined

II. $\lim\limits_{x \to c} f(x)$ exists.

III. $f(c) = \lim\limits_{x \to c} f(x)$

<u>Show</u> (THREE STEPS) that each of the following functions is either continuous or discontinuous at the given value of *x*.

2. $f(x) = x + 5$ at $x = 1$	3. $f(x) = x^2 + 2x - 1$ at $x = 0$
4. $f(x) = \dfrac{x^2 - 16}{x - 4}$ at $x = 4$	5. $f(x) = \dfrac{x^2 - 25}{x + 5}$ at $x = 5$
6. $f(x) = \dfrac{1}{x}$ at $x = 3$	7. $f(x) = \dfrac{3x - 1}{2x + 6}$ at $x = -3$

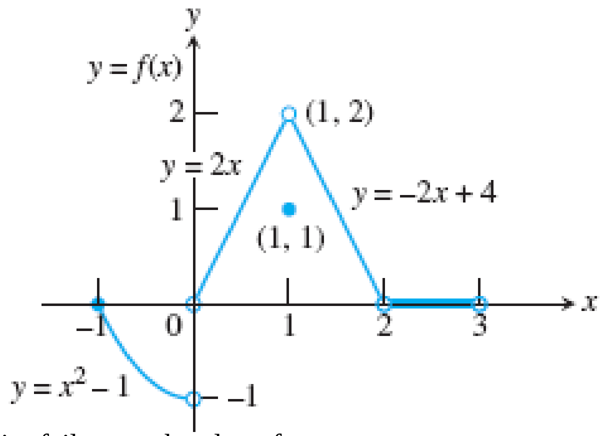


8. Determine the intervals on which the graph of $f(x)$, shown above, is continuous

State the open interval(s) on which each function is continuous.

9. $f(x) = x^2 + 2$	10. $f(x) = \frac{1}{x}$
11. $f(x) = \frac{x^2+1}{x-1}$	12. $f(x) = \frac{x^2-3x+2}{x^2+4x-5}$

$$f(x) = \begin{cases} x^2 - 1, & -1 \le x < 0 \\ 2x, & 0 < x < 1 \\ 1, & x = 1 \\ -2x+4, & 1 < x < 2 \\ 0, & 2 < x < 3 \end{cases}$$



13. Given $f(x)$ and the graph of $f(x)$, state why continuity fails at each value of x.

14. State the interval(s) on which $f(x)$ is continuous.

AP Calculus AB – Worksheet 14 **Continuity - Removable**

For questions 1 & 2,

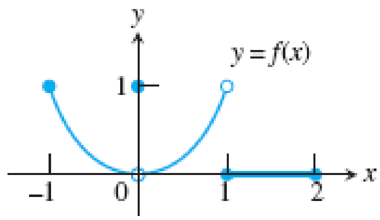
a) Determine the x-coordinate of each discontinuity on the graph of $f(x)$.

b) Identify each discontinuity as either **removable** or **jump**.

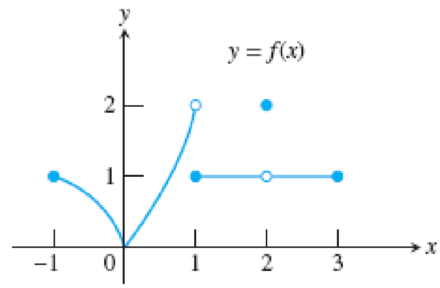
c) Evaluate the limit at each discontinuity.

d) State the interval(s) on which $f(x)$ is continuous.

1.



2.



For questions 3-5, answer the following:

a) Determine the x-coordinates of any discontinuities on the graph of $f(x)$.

b) Identify the discontinuities as either **infinite** or **removable**.

c) Evaluate the limit at each **removable** discontinuity.

d) State the interval(s) on which $f(x)$ is continuous.

e) Write an extended function of $g(x)$ that is continuous.

3. $f(x)=\dfrac{x-2}{x^2-5x+6}$	4. $f(x)=\dfrac{x^2-4}{x+2}$	5. $f(x)=\dfrac{x^2+x-12}{x^2+6x+8}$

6. Find the value of a that makes $f(x)=\begin{cases} 2x^2-4, & x<2 \\ ax+3, & x\geq 2 \end{cases}$ continuous on the entire interval.
7. Find the value of a that makes $f(x)=\begin{cases} ax^2+2, & x<3 \\ 4x-1, & x\geq 3 \end{cases}$ continuous on the entire interval.
8. Find the value of k that makes $f(x)=\begin{cases} \dfrac{(2x+1)(x-2)}{x-2}, & x\neq 2 \\ k & x=2 \end{cases}$ continuous on the entire interval.

Evaluate each limit

9. $\lim\limits_{x\to\infty}\dfrac{x^{99}}{e^x}$	10. $\lim\limits_{x\to\infty}\dfrac{e^x}{\ln x}$	11. $\lim\limits_{x\to\infty}\dfrac{\ln x}{x^{99}}$	12. $\lim\limits_{x\to-\infty}\dfrac{x^{99}}{e^x}$

Each of the following has a removable discontinuity. Find an extended function, $g(x)$, that removes the discontinuity.

1) $f(x)=\frac{x^2-5x+6}{x-3}$	2) $f(x)=\frac{x^2-5}{x-\sqrt{5}}$	3) $f(x)=\frac{x^3+8}{x+2}$

4) Find each limit

a) $\lim_{x\to -3^-}\frac{-3x}{x+3}$ b) $\lim_{x\to -3^+}\frac{-3x}{x+3}$

5) Find the vertical asymptote on the graph of $f(x)=\frac{x^2-10x}{x+9}$. Describe the behavior of $f(x)$ to the left and right of the vertical asymptote.

For problems 6-7,

a) Determine the *x*-coordinate of the discontinuities on the graph of $f(x)$. Identify the discontinuities as either infinite or removable.
b) Use limits to describe the behavior of $f(x)$ near any removable discontinuities.
c) State the interval(s) on which $f(x)$ is continuous.
d) Identify any vertical asymptotes on the graph of $f(x)$. Use limits to describe the behavior of $f(x)$ near the vertical asymptote.
e) Use limits to identify any horizontal asymptotes on the graph of $f(x)$.
f) Draw an accurate graph of $f(x)$.

6) $f(x)=\frac{x^2-4}{x^2+3x+2}$	7) $f(x)=\frac{2x^2-5x+3}{x^2+6x-7}$

8) Sketch the graph of any function such that:

$\lim_{x\to -\infty} f(x)=-\infty$

$f(1)=3$

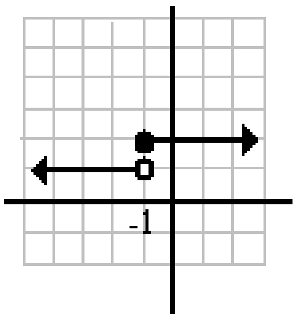
$\lim_{x\to 1} f(x)=-2$

$\lim_{x\to \infty} f(x)=5$

To live for results would be to sentence myself to continuous frustration. My only sure reward is in my actions and not from them. – Hugh Prather

For problems 1-4, use the graph to test the function for continuity at the indicated value of *x*.

1.

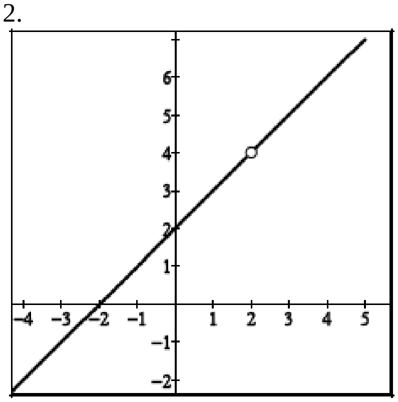


1a) Explain why continuity fails at $x=-1$.

1b) What kind of discontinuity does $f(x)$ have?

1c) On what open interval(s) is $f(x)$ continuous?

2.

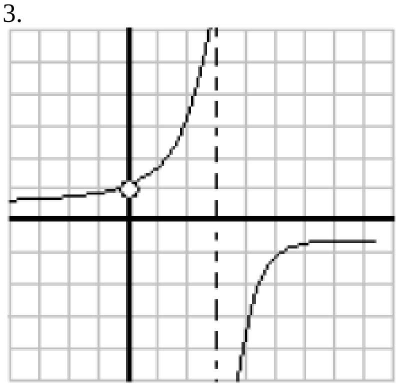


2a) Explain why continuity fails at $x=2$.

2b) What kind of discontinuity does $f(x)$ have?

2c) On what open interval(s) is $f(x)$ continuous?

3.

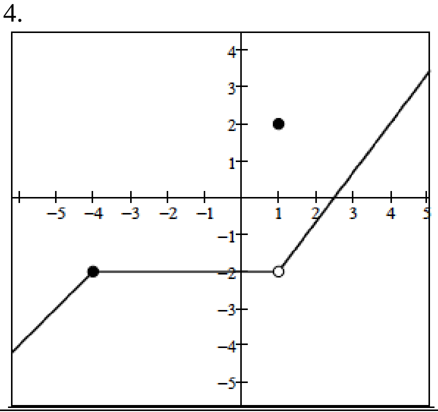


3a) Explain why continuity fails at $x=3$.

3b) What kind of discontinuity does $f(x)$ have at $x=3$?

3c) On what open interval(s) is $f(x)$ continuous?

4.



4a) Explain why continuity fails at $x=1$.

4b) What kind of discontinuity does $f(x)$ have?

4c) On what open interval(s) is $f(x)$ continuous?

Determine whether $f(x)$ is continuous at the given value of *x*.

5) $f(x)=\begin{cases}\sin \pi x, & x\le 2\\ x^2+3x-9, & x>2\end{cases},\quad x=2$	6) $f(x)=\begin{cases}-2x+3, & x<1\\ x^2, & x\ge 1\end{cases},\quad x=1$

Find the constant *a*, or the constants *a* and *b*, such that the function is continuous on the entire number line.

7) $f(x)=\begin{cases}x^3, & x\le 2\\ ax^2, & x>2\end{cases}$	8) $f(x)=\begin{cases}4-x^2, & x<-1\\ ax^2-1, & x\ge -1\end{cases}$
9) $f(x)=\begin{cases}2, & x\le -1\\ ax+b, & -1<x<3\\ -2, & x\ge 3\end{cases}$	10) $f(x)=\begin{cases}x^2+3x, & x<2\\ a, & x=2\\ 7x-4, & x>2\end{cases}$

In 1-3, verify that the Intermediate Value Theorem guarantees that there is a zero in the interval for the given function.

1) $f(x)=\frac{1}{16}x^4-x^3+3$; $[1,2]$

2) $f(x)=x^3+3x-2$; $[-2,1]$

3) $f(x)=x^2-x-\cos x$; $[0,\pi]$

In 4-6, verify that the Intermediate Value Theorem applies to the indicated interval and find the value of *c* guaranteed by the theorem. No calculator is permitted on these problems.

4) $f(x)=x^2+x-1$
$f(c)=11$ in $[0,5]$

5) $f(x)=x^2-6x+8$
$f(c)=5$ in $[0,3]$

6) $f(x)=x^3-x^2+x-2$
$f(c)=4$ in $[0,3]$

7 Given selected values of the continuous function $h(x)$, what is the fewest number of times $h(x)=43$ in the interval $[0,30]$? Explain your reasoning.

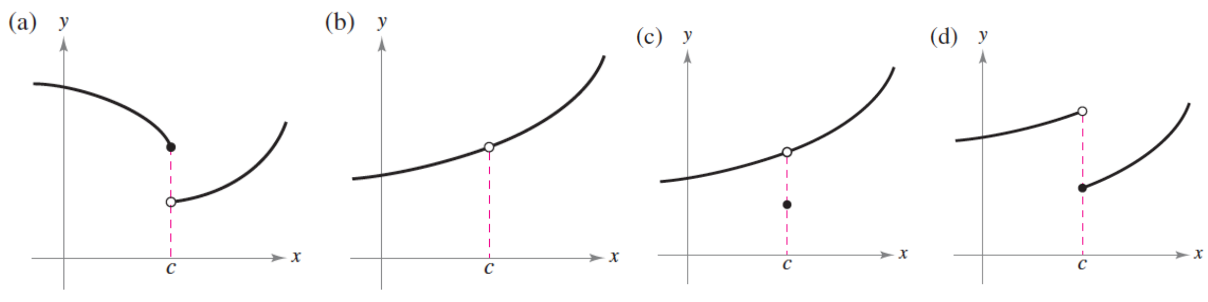
x	0	5	10	15	20	25	30
$h(x)$	100	40	40	110	30	10	50

8 Water is pumped into a tank at a rate modeled by $W(t)=2000e^{\frac{-t^2}{20}}$ liters per hour for $0\le t\le 8$, where *t* is measured in hours. Water is removed from the same tank at a rate modeled by $R(t)$ liters per hour, where R is continuous and decreasing on $0\le t\le 8$. Selected values of $R(t)$ are shown in the table below.

t (hours)	0	1	3	6	8
$R(t)$ (liters/hour)	1340	1190	950	740	700

For $0\le t\le 8$, is there a time *t* when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain your reasoning.

9 For each of the graphs below state how continuity is destroyed at $x=c$.

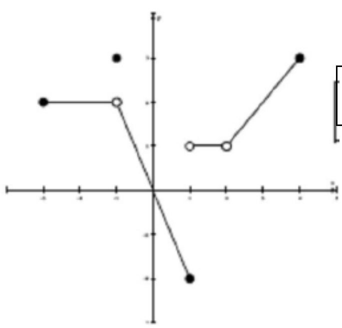


10 Determine whether $f(x)$ is continuous at $x=-1$ for
$$f(x)=\begin{cases}\frac{1}{x}, & x\le -1\\ \frac{x-1}{2}, & -1<x<1\\ \sqrt{x}, & x\ge 1\end{cases}$$

AP Calculus AB - Worksheet 18 **Limits and Their Properties Review**

1	$\lim_{x\to\frac{\pi}{2}} \frac{1+\sin x}{1-\cos x}$	2	$\lim_{x\to\infty} \frac{\cos 2x}{x^2}$
3	$\lim_{x\to\infty} \left(\frac{6x^3-5x}{x^2+4x^3}\right)$	4	$\lim_{x\to\infty} \frac{x^2+x^4}{x^2+x^6}$
5	$\lim_{x\to-\infty} \frac{8x^3-5x}{x^2-3x}$	6	$\lim_{x\to 4^-} \frac{5}{x-4}$
7	$\lim_{x\to 2} \frac{4x^3-32}{5x^2-20}$	8	$\lim_{x\to-\infty} \frac{1}{1+e^x}$
9	$\lim_{x\to-\infty} \frac{e^x}{4x^3-3}$	10	$\lim_{x\to 0} \frac{\sin 3x}{7x}$
11	Sketch a graph of a function such that: $f(3)=2$ $\quad$ $\lim_{x\to 3} f(x)=-1$ $\lim_{x\to -2^-} f(x)=-\infty$ $\quad$ $\lim_{x\to -2^+} f(x)=\infty$ $\lim_{x\to -\infty} f(x)=5$ $\quad$ $\lim_{x\to \infty} f(x)=-\infty$	12	Find the value of a that will make $g(x)$ continuous. $g(x)=\begin{cases} ax+3, & x\le 1 \\ (x+a)^2-10, & x>1 \end{cases}$

Use the following graph of $f(x)$ to answer the next five questions. If the limit does not exist, state why.



13 $\lim_{x\to 1^+} f(x)$	14 $\lim_{x\to 1^-} f(x)$	15 $\lim_{x\to 1} f(x)$	16 $\lim_{x\to -1} f(x)$	17 $\lim_{x\to 2} f(x)$

18	Evaluate $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ for $f(x)=2x^2-7x$
19	If $f(x)=\begin{cases} 2x-1, & x\le 1 \\ -3x+1, & x>1 \end{cases}$, determine if $f(x)$ continuous at $x=1$.
20	If $f(x)=\begin{cases} \frac{x^2+6x+8}{x+2}, & x\ne -2 \\ 2, & x=-2 \end{cases}$, determine if $f(x)$ is continuous at $x=-2$.
21	Find an extended function of $f(x)=\frac{x^2-16}{x-4}$ that is continuous.

Answers

Worksheet 8

1)

a) -10	b) 7	c) 3	d) $-\frac{1}{c}$
e) 24	f) $\frac{5}{9}$	g) $-\frac{2}{5}$	h) 4

2)

a) 3	b) 2	c) 0
d) 8	e) $\frac{1}{2}$	f) -2

3)

a) 10	b) 0	c) 25	d) -5

4) -9
5) 7

Worksheet 9

1) -7	2) 5	3) $-\sqrt{2}$	4) $-\frac{1}{2}$	5) 1	6) 1	7) 0
8) DNE	9) 3	10) $\frac{2}{5}$	11) DNE	12) a) 3 b) 3 c) 3 d) undefined	13) a) 15 b) 3 c) DNE d) 3	

Worksheet 10

1) 12	**2)** $\frac{6}{7}$	**3)** $\frac{20}{3}$
4) -2	**5)** 4	**6)** 1
7) -5	**8)** $\frac{1}{8}$	**9)** DNE
10) $\frac{7}{16}$	**11)** 20	**12)** 18
13) DNE	**14)** -36	**15)** $a=-4$

Worksheet 12

Quiz Review

1) 2; 2; 2; 1	2) 2; 4; DNE; 4	3) 11	4) 3
5) $\frac{3}{4}$	6) 2	7) -1	8) $\frac{7}{2}$

Limits at Infinity

1) $-\infty$	2) ∞	3) 8	4) 9
5) 1	6) 1	7) $\sqrt{2}$	8) $-\sqrt{2}$
9) 0	10) $-\infty$	11) 0	12) -1
13) I and III	14) a) $y=5$ b) $y=-3$ c) $y=1$; $y=-1$		

Worksheet 18

1. 2	2. 0	3. $\frac{3}{2}$	4. 0	5. DNE	6. $-\infty$	7. $\frac{12}{5}$
8. 1	9. 0	10. $\frac{3}{7}$	11. Answers will vary	12. -4, 3	13. 1	14. -2
15. DNE	16. 2	17. 1	18. $4x-7$			

19.

I. $f(1)=1$

II. $\left[\lim_{x\to 1^+} f(x)=1 \neq \lim_{x\to 1^-} f(x)=-2\right]$

$\lim_{x\to 1}$ DNE

$\therefore\ f(x)$ is not continuous at $x=1$

20.

I. $f(-2)=2$

II. $\lim_{x\to -2}\frac{x^2+6x+8}{x+2}=\lim_{x\to -2}(x+4)=2$

III. $f(-2)=\lim_{x\to -2} f(x)=2$

$\therefore\ f(x)$ is continuous at $x=-2$

21.

$f(x)$ is not continuous at $x=4$

$$\lim_{x\to 4}\frac{x^2-16}{x-4}=\lim_{x\to 4}\frac{(x-4)(x+4)}{(x-4)}=\lim_{x\to 4}(x+4)$$

$$g(x)=\begin{cases}\dfrac{x^2-16}{x-4}, & x\neq 4\\ 8, & x=4\end{cases}$$